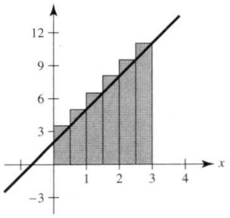
Section 4.2 (page 267)

1. 75 **3.** $\frac{158}{85}$ **5.** $4c$ **7.** $\sum_{i=1}^{11} \frac{1}{5i}$ **9.** $\sum_{j=1}^{6} \left[7\left(\frac{j}{6}\right) + 5\right]$

11. $\frac{2}{n}\sum_{i=1}^{n}\left[\left(\frac{2i}{n}\right)^3 - \left(\frac{2i}{n}\right)\right]$ **13.** $\frac{3}{n}\sum_{i=1}^{n}\left[2\left(1 + \frac{3i}{n}\right)^2\right]$ **15.** 84

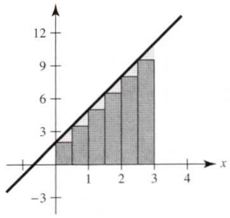
17. 1200 **19.** 2470 **21.** 12,040 **23.** 2930

25. (a)



Area ≈ 21.75

(b)



Area ≈ 17.25

27. 13 < (Area of region) < 15

29. 55 < (Area of region) < 74.5

31. 0.7908 < (Area of region) < 1.1835

33. The area of the shaded region falls between 12.5 square units and 16.5 square units.

35. The area of the shaded region falls between 7 square units and 11 square units.

37. $\frac{81}{4}$ **39.** 9 **41.** $A \approx S \approx 0.768$, $A \approx s \approx 0.518$ **43.** $A \approx S \approx 0.746$, $A \approx s \approx 0.646$

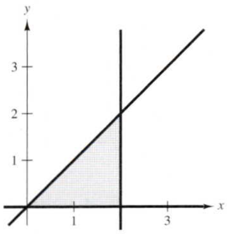
45. $(n + 2)/n$
$n = 10$: $S = 1.2$
$n = 100$: $S = 1.02$
$n = 1000$: $S = 1.002$
$n = 10{,}000$: $S = 1.0002$

47. $[2(n + 1)(n - 1)]/n^2$
$n = 10$: $S = 1.98$
$n = 100$: $S = 1.9998$
$n = 1000$: $S = 1.999998$
$n = 10{,}000$: $S = 1.99999998$

49. $\lim_{n\to\infty}\left[\frac{12(n + 1)}{n}\right] = 12$ **51.** $\lim_{n\to\infty}\frac{1}{6}\left(\frac{2n^3 - 3n^2 + n}{n^3}\right) = \frac{1}{3}$

53. $\lim_{n\to\infty}[(3n + 1)/n] = 3$

55. (a)



(b) $\Delta x = (2 - 0)/n = 2/n$

(c) $s(n) = \sum_{i=1}^{n} f(x_{i-1})\,\Delta x = \sum_{i=1}^{n}[(i - 1)(2/n)](2/n)$

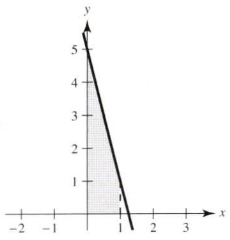
(d) $S(n) = \sum_{i=1}^{n} f(x_i)\,\Delta x = \sum_{i=1}^{n}[i(2/n)](2/n)$

(e)

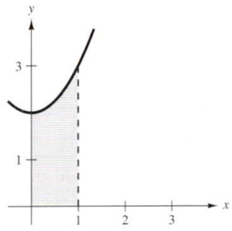
n	5	10	50	100
$s(n)$	1.6	1.8	1.96	1.98
$S(n)$	2.4	2.2	2.04	2.02

(f) $\lim_{n\to\infty}\sum_{i=1}^{n}[(i - 1)(2/n)](2/n) = 2$; $\lim_{n\to\infty}\sum_{i=1}^{n}[i(2/n)](2/n) = 2$

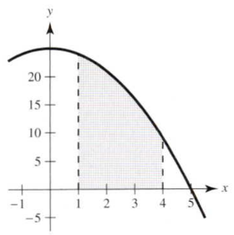
57. $A = 3$



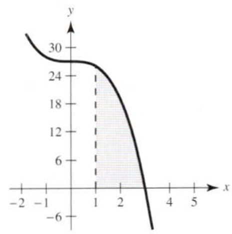
59. $A = \frac{7}{3}$



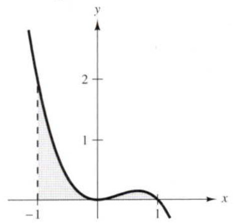
61. $A = 54$



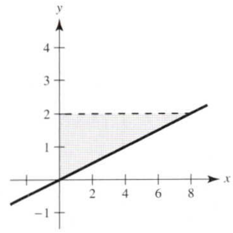
63. $A = 34$



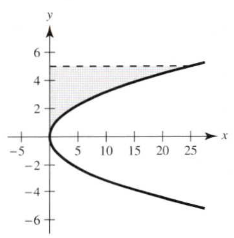
65. $A = \frac{2}{3}$



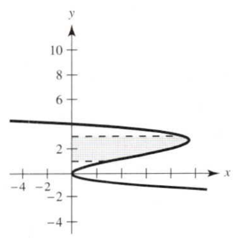
67. $A = 8$



69. $A = \frac{125}{3}$



71. $A = \frac{44}{3}$



73. $\frac{69}{8}$ **75.** 0.345

77.

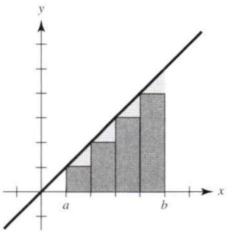
n	4	8	12	16	20
Approximate Area	5.3838	5.3523	5.3439	5.3403	5.3384

79.

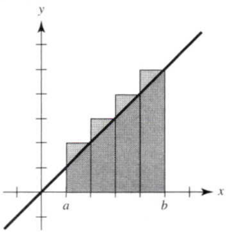
n	4	8	12	16	20
Approximate Area	2.2223	2.2387	2.2418	2.2430	2.2435

81. b

83. You can use the line $y = x$ bounded by $x = a$ and $x = b$. The sum of the areas of the inscribed rectangles in the figure below is the lower sum.

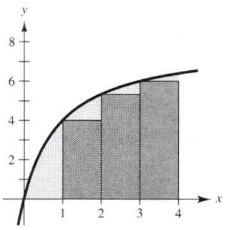


The sum of the areas of the circumscribed rectangles in the figure below is the upper sum.



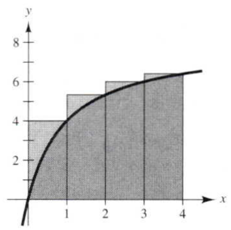
The rectangles in the first graph do not contain all of the area of the region, and the rectangles in the second graph cover more than the area of the region. The exact value of the area lies between these two sums.

85. (a)



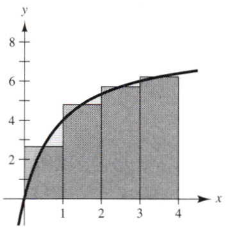
$s(4) = \frac{46}{3}$

(b)



$S(4) = \frac{326}{15}$

(c)



$M(4) = \frac{6112}{315}$

(d) Proof

(e)

n	4	8	20	100	200
$s(n)$	15.333	17.368	18.459	18.995	19.060
$S(n)$	21.733	20.568	19.739	19.251	19.188
$M(n)$	19.403	19.201	19.137	19.125	19.125

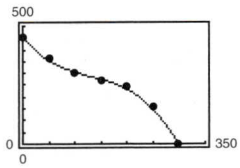
(f) Because f is an increasing function, $s(n)$ is always increasing and $S(n)$ is always decreasing.

87. True

89. Suppose there are n rows and $n + 1$ columns. The stars on the left total $1 + 2 + \cdots + n$, as do the stars on the right. There are $n(n + 1)$ stars in total. So, $2[1 + 2 + \cdots + n] = n(n + 1)$ and $1 + 2 + \cdots + n = [n(n + 1)]/2$.

91. (a) $y = (-4.09 \times 10^{-5})x^3 + 0.016x^2 - 2.67x + 452.9$

(b)



(c) $76{,}897.5 \text{ ft}^2$

93. Proof